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NUMERICAL METHODS IV INTERPOLATION AND SHAPE FUNCTIONS

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SOME MATHEMATICAL SYMBOLS

\forall	<i>for</i>
\exists	<i>exist(s)</i>
\setminus	<i>except, excluding</i>
$ $	<i>with property</i>
\wedge	<i>and</i> (binary logical operator)
\vee	<i>or</i> (binary logical operator)
$\{ \}$	set of elements or vector
\in	element of a set
\cup	union of two (or more) sets
\cap	common elements of two (or more) sets
$\langle \rangle$	inner product
Σ	sum of
Π	product of
∞	infinity

Examples: Usage of some mathematical symbols.

$\forall(i, j)|i, j = 1, 2, 3$ means: for pair (i, j) with property $i \in \{1, 2, 3\}$, $j \in \{1, 2, 3\}$.

Inner product on a discrete domain

$$\langle a, b \rangle = \sum_{n=1}^N w^n a(x^n) b(x^n) \quad (w^n \text{ are weighting coefficients}).$$

Note: Upper index is not always an exponent, i.e., it is an exponent if and only if it is clearly stated. In the previous example x^n denotes n^{th} point in a discrete domain and it is not an n^{th} power of x .