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REFERENCES

- [1] Hildebrand, F. B., *Introduction to Numerical Analysis*, Dover Publications, ISBN 0-486-65363-3, New York, 1987.
- [2] Householder, A.S., *Principles of Numerical Analysis*, Dover Publications, 1981.
- [3] Berrut, J. P.; Trefethen, L. N., *Barycentric Lagrange interpolation*, SIAM Review 46: 501–517, 2004.
- [4] Runge, C., *Über empirische Funktionen und die Interpolation zwischen äquidistanten Ordinaten*, Zeitschrift für Mathematik und Physik 46: 224–243, 1901.
- [5] Obsieger, B., *Numerical Methods III, Approximation of Functions*, University of Rijeka, Faculty of Engineering, ISBN 978-953-6326-68-6, Rijeka, 2011.
- [6] Chapra, S.C., Canale, R.P., *Numerical Methods for Engineers*, McGraw Hill, ISBN 978-007-126759-5, New York, 2010.
- [7] Zienkiewicz, O. C., *The Finite Element Method in Engineering Science*, McGraw-Hill, London, 1971.

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